meap

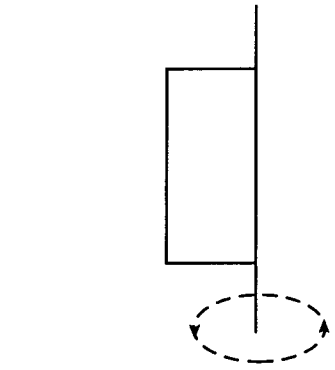
Michigan Educational Assessment Program

High School Test in Mathematics

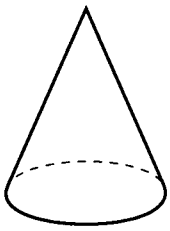
Released Items

Spring 2000

1 If you attach a rectangle to a wire as shown in the diagram below, and rotate it by spinning the wire between your fingers, you would generate which of the following 3-dimensional figures?



A



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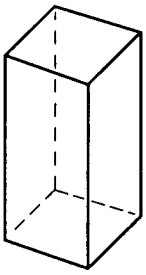
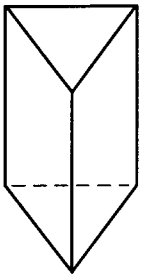


C



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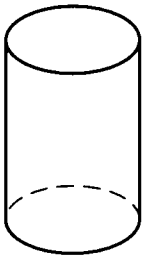
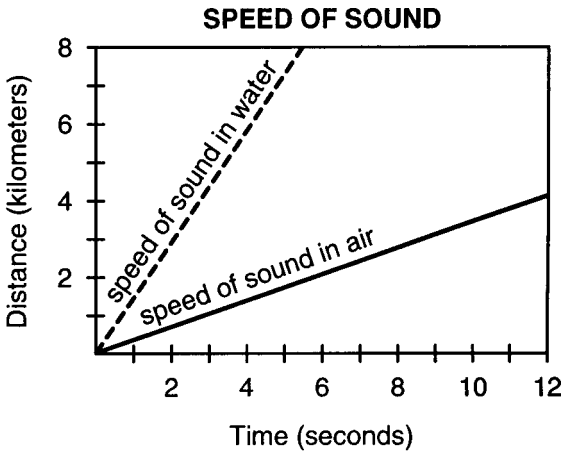


3



The graph shows the speed of sound in air compared with the speed of sound in water. Based on the graph, which statement is **TRUE**?

A Sound travels faster in air than in water.

B Sound travels faster in water than in air.

C The distance sound travels in water is determined by the time it travels in air.

D The speed of sound is determined by multiplying the distance it travels by the time.

4 After the homecoming dance, the student clean-up committee collected all the empty drink containers. There were 25 more 10¢ deposit containers than 5¢ deposit containers. Which expression represents the monetary value of the empty drink containers?

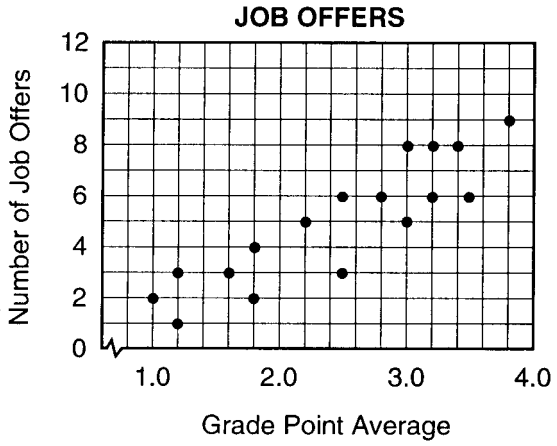
A $0.15n$

B $0.15(n+25)$

C $0.10n+0.05(n+25)$

D $0.05n+0.10(n+25)$

5 The scatter plot below shows the number of job offers and grade point averages for the students in an economics class. Which of the following would describe the best-fit line for the given data points?



A a horizontal line

B a vertical line

C a line with a negative slope

D a line with a positive slope

6 A pizza parlor decided to give away one of six different posters of the Detroit Red Wings hockey players with each pizza sold. To simulate how many pizzas a person would have to buy to collect all six posters, the students in a math class decided to assign one number to each of the six posters and roll a number cube, numbered 1 through 6. The results of the first 20 rolls are listed below in the order in which they were rolled.

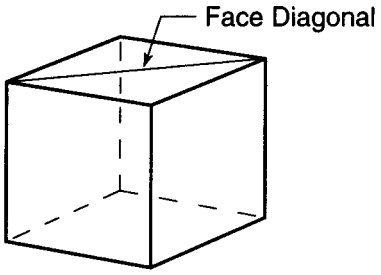
4, 1, 1, 4, 6, 3, 3, 2, 2, 4,
5, 1, 6, 2, 5, 1, 5, 3, 2, 1

Based on this simulation, what is the minimum number of pizzas a person would have to buy to collect all six posters? (Assume there is an equally likely chance of getting any poster.)

A 6 **B** 8

C 11 **D** 20

12 A vertical cut is made through the cube along the diagonal of the top face as shown below. Which two identical solids will be produced?



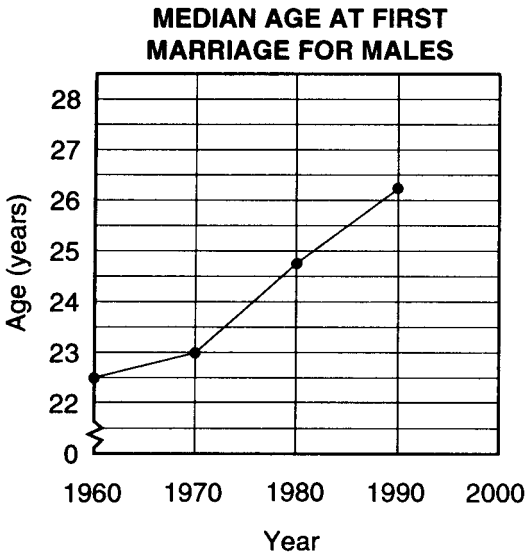
A rectangular pyramids

B rectangular prisms

C triangular pyramids

D triangular prisms

13 If the trend shown on this graph continues, at what median age would a man marry for the first time in the year 2000?



A younger than 26.0

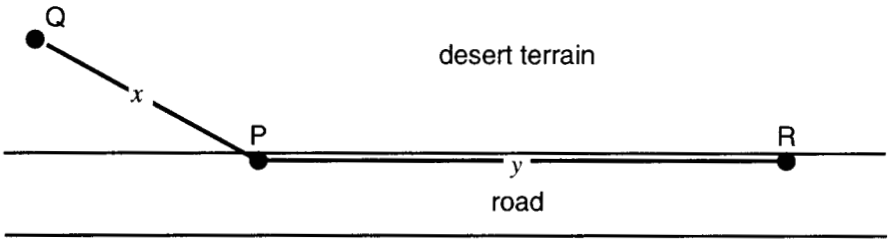
B between 26.0 and 26.9

C between 27.0 and 28.0

D older than 28.0

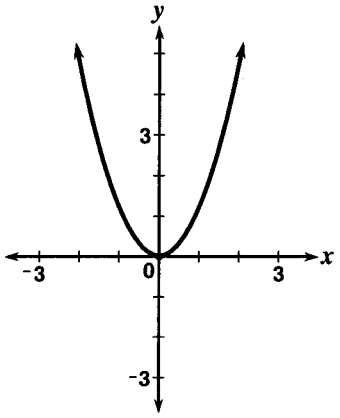
17 Given: A vehicle can travel across the desert at 20 mph. Its road speed is 50 mph.
The distance from Q to P is x miles.
The distance from P to R is y miles.

Which expression represents the time it would take to travel from Q to R through P ?



A $20x + 50y$ **B** $50x + 20y$ **C** $\frac{x}{20} + \frac{y}{50}$ **D** $\frac{(x+y)}{(20+50)}$

19 Look at the graph below. Which table **BEST** represents the graph?



A

x	y
–2	4
–1	1
0	0
1	1

B

x	y
–2	4
–1	1
0	0
1	–1

C

x	y
–2	–4
–1	1
0	0
1	1

D

x	y
–2	–4
–1	1
0	0
1	–1

21 Two automobiles with different fuel efficiencies both travel 12,000 miles per year. Car A averages 40 miles per gallon and Car B averages 24 miles per gallon. How many fewer gallons of gasoline would Car A require per year?

A 200 **B** 300

C 500 **D** 750

22 The students in an economics class made the following table to show the number of television sets in their homes.

Number of Television Sets	Number of Students Responding
3 or more	6
2	13
1	9
0	2

If the students in the economics class are representative of the students in the entire school, how many of the school's 600 students would you predict have 3 or more television sets in their homes?

A 60 **B** 120

C 180 **D** 200

23 The total number of bacteria in a petri dish is 2,598,960. What is this number written in scientific notation?

A 2.59896×10^{-6}

B 2.59896×10^{6}

C 259896×10^{-1}

D 259896×10^{1}

25 A rectangle has $(0,12)$, $(3,16)$, and $(12,3)$ as coordinates of three of its vertices. Which coordinates represent the fourth vertex of the rectangle?

A $(15,-1)$ **B** $(15,7)$

C $(16,6)$ **D** $(8,0)$

26 A rectangle that is **NOT** a square has side lengths that are whole numbers. If the perimeter of the rectangle measures 8 cm, what is the area of the rectangle?

A $3\ cm^2$ **B** $4\ cm^2$

C $12\ cm^2$ **D** $15\ cm^2$

29 The school paper charges \$1.50 for an ad, plus 25 cents per word. Carl wants to spend only \$12.00 on the ad. What is the maximum number of words that can be in his ad?

A 18 **B** 42

C 48 **D** 54

32 Three points lie on the same line. If the coordinates of two of the points are $(0,-6)$ and $(4,0)$, which of the following coordinates could represent the third point?

A $(2,-3)$ **B** $(2,-2)$

C $(3,-2)$ **D** $(1,-5)$

(2 Points)

35 Vivian is graduating from high school next year and is considering attending college to study engineering. She read an article that listed the average starting salary as $40,000 among 100 engineering students who graduated last year. The article noted that the average included one graduate who was also a basketball star, who had signed a $1.5 million per year professional contract. Explain why the average starting salary might be misleading. Include calculations or equations to support your explanation.

35a) Sample Response

The average starting salary might be misleading because the addition of 1.5 million to the total salary amount will increase the average.

This is because...

1. ➢ the actual starting salary is $25,252 for the 99 students, compared to $40,000 for all 100.
   100 graduates x $40,000/graduate = $4,000,000
   $4,000,000 – $1,500,000 = $2,500,000
   $2,500,000 ÷ 99 graduates ≈ $25,252/graduate
   OR
2. ➢ the actual starting salary is $25,252 for the 99 students, compared to $40,000 for all 100.
   (x + 1,500,000) ÷ 100 = 40,000
   x + 1,500,000 = 4,000,000
   x = 2,500,000 2,500,000 ÷ 99 ≈ $25,252
   OR
3. ➢ there is a $15,000 difference between the actual salary and $40,000.
   $40,000 - $25,000 = $ 15,000
   OR
4. ➢ an outlier will cause the average to be skewed (use of hypothetical numbers).
   1 + 2 + 3 + 4 = 10 10 ÷ 4 = 2.5
   1 + 2 + 3 + 4 + 500 = 510 510 ÷ 5 = 102
   OR
5. ➢ if starting salary actually is $40,000, then 99 x $40,000 = $3,960,000
   3,960,000 + 1,500,000 = 5,460,000 5,460,000 ÷ 100 = 54,600 which is much higher than 40,000.
   OR
6. ➢ the basketball player's salary is 37.5% of the total salary while each of the other 99 students only account for 0.6% of the total salary.
   1,500,000 ÷ 4,000,000 = .375 x 100 = 37.5%
   25,000 ÷ 4,000,000 = .006 x 100 = 0.6%

35b) Scoring Rubric

2 points

A correct explanation is given with correct calculations or equations as support.
Note: Support may be given in the form of verbal calculations.

1 point

A correct explanation is given without substantive support.

OR

The correct mathematical process is used to support the explanation, but an explanation is either incorrect or not given.

0 points

An attempt is made, but the approach is invalid.

35c) Student Response 1

The average starting salary would be misleading because it would be greater than it should be. If you add a number that is significantly greater than the others to an average, the resulting average is not an accurate portrayal of the other numbers in the group.

For example, the average of 2, 3, 5, 7, 8, 9 is

$$\frac{2+3+5+7+8+9}{6} = \frac{34}{6} = 5.6$$

But the average of 2, 3, 5, 7, 8, 40 is

$$\frac{2+3+5+7+8+40}{6} = \frac{65}{6} = 10.8$$

40 is much greater than the rest of the numbers, and the results are misleading. From these results, a person would conclude that the numbers in the group that averaged 10.8 are greater than they actually are.

Score Point: 2

This response gives a correct explanation (average would be greater) with a correct hypothetical calculation as support (Sample 4).

35d) Student Response 2

The average salary is misleading because the salary of that basketball player brings it up $15,000. The average salary of the other 99 graduates was only $25,000

Score Point: 2

This response gives a minimally correct explanation (basketball player brings the average up) with correct calculations ($15,000 compared to $25,000) as support (Sample 3).

35e) Student Response 3

The average starting salary might be misleading because the averge is the salary of all 100 graduates divided by number of graduates (100). Because one of the graduates salary was $1.5 million per year because he/she was a basketball player, that 1.5 when added into the average / mean boosts up the average to $40,000.

Score Point: 1

This response gives a correct explanation (boosts up the average), but does not provide correct calculations as support. The calculations given just explain how to compute an average.

35f) Student Response 4

It may be misleading because the article probably included the basketball players 1.5 million dollar contract into the average.

An equation might be

$$\text{Average Salary} = \frac{X + 1.5\text{million}}{60}$$

X = real average salary

Score Point: 0

This response gives neither a correct explanation nor a correct calculation as support. Just stating how to calculate an average does not receive credit.

(4 Points)

36 Through July 31, a baseball team won 42 games and lost 56. They have 52 games remaining on their schedule. In order to win the league title, a team must usually end the season with a record of **AT LEAST** 0.600. (The record expressed as a decimal equals the number of wins divided by the number of games played.)

A If the team is to finish with a record of 0.600, explain how you would determine the number of remaining games they must win. How many more games must the team win to have a record of 0.600?

ANSWER THIS ITEM IN YOUR ANSWER BOOKLET.
NOTHING WRITTEN IN THE SPACE BELOW WILL BE SCORED.

B What is the **HIGHEST** record for the complete year the team can achieve with the remaining number of games? What is the **LOWEST** record? Express your answers as decimals by dividing the number of wins by the number of games played.

36a) Sample Response

Part A

Method 1

x = least number of games that must be won

Total number of games = 42 + 56 + 52 = 150

Winning Average equation: $\frac{\text{Number of wins}}{\text{Total games}} = \frac{42 + x}{150} = 0.600$

Solve for x: $\frac{42 + x}{150} = 0.600$

$42 + x = 90$

$x = 90 - 42 = 48$

The team must win at least 48 more games to have a record of at least 0.600.

Method 2

Total number of games = 42 + 56 + 52 = 150

Number of games needed to win to achieve 0.600:
150 x 0.600 = 90

Number of games left to win = 90 – 42 = 48

The team must win at least 48 more games to have a record of at least 0.600.

Part B

Highest possible record (i.e. team wins all remaining games):
$\frac{\text{\# wins}}{\text{Total}} = \frac{42 + 52}{150} = 0.627$ (**OR** 0.6, 0.62, 0.63, 0.626, $0.62\overline{6}$)

Lowest possible record (i.e. team loses all remaining games):
$\frac{\text{\# wins}}{\text{Total}} = \frac{42 + 0}{150} = 0.280$

36b) Scoring Rubric

Part A (2 points)

2 The correct number of games left to be won (48) is given and work is shown to explain how the answer was determined.

1 The correct number of games left to be won (48) is given, but either insufficient, incorrect, or no work is shown to explain the answer.

OR

A correct method is used to arrive at the number of games left to be won, but a computational error produces an incorrect answer or no answer is given.

0 The number of games left to be won is incorrect and an incorrect method is used to arrive at the answer.

Part B (2 points)

2 Both the highest possible record and lowest possible record are correctly given as decimals.

1 Only one of the records is correctly given as a decimal.

OR

The highest and lowest possible records are correctly given, but in a form other than as decimals (e.g. highest: 94 wins or 627; lowest: 42 wins or 280).

0 Neither of the records are given.

OR

One of the records is given in a form other than a decimal.

Note: If the student miscalculates the total number of games in Part A and **correctly** uses this miscalculation in Part B, credit can be given in Part B.

36c) Student Response 1

A. The team must win 90 games for the whole season in order to finish with a record of .600. Which means they must win 48 out of their 52 games left.

$\frac{90}{150} = .600$

In order to find out how many games this team needs to finish with a record of .600 you have to divide 150 the total number of games played by the unknown variable X and equal it to the record .600

$\frac{X}{150} = .600$

B. $\frac{94}{150} = .63$ highest record

$\frac{42}{150} = .28$ this would be their average if they didn't win any of their 52 games left.

Score Point: 4

This response gives the correct number of games left to be won (48) in Part A with work shown, and gives the highest (.63) and lowest (.28) records expressed as decimals in Part B.

36d) Student Response 2

A. $98 + 52 = 150$

$.600 \times 150 = 90$

$90 - 42 =$ (48 more wins are needed)

B. highest = .626

lowest = .28

Score Point: 4

This response gives the correct number of games left to be won (48) in Part A with work shown, and gives the highest (.626) and lowest (.28) records expressed as decimals in Part B.

36e) Student Response 3

Right now they have a score of .428. In order to win, they must win .172 more games. They must win 48 of the 52 games left. $x + 42 = .6$

B) $42 + 52 = \frac{94}{150} = .626$

$42 + 0 = \frac{42}{150} = .28$

Score Point: 3

This response gives the correct number of games left to be won (48) in Part A with insufficient work shown, and gives the highest (.626) and lowest (.28) records expressed as decimals in Part B.

36f) Student Response 4

won lost
42 + 56 = 98 games played

games played games left
98 + 52 = 150 games per season

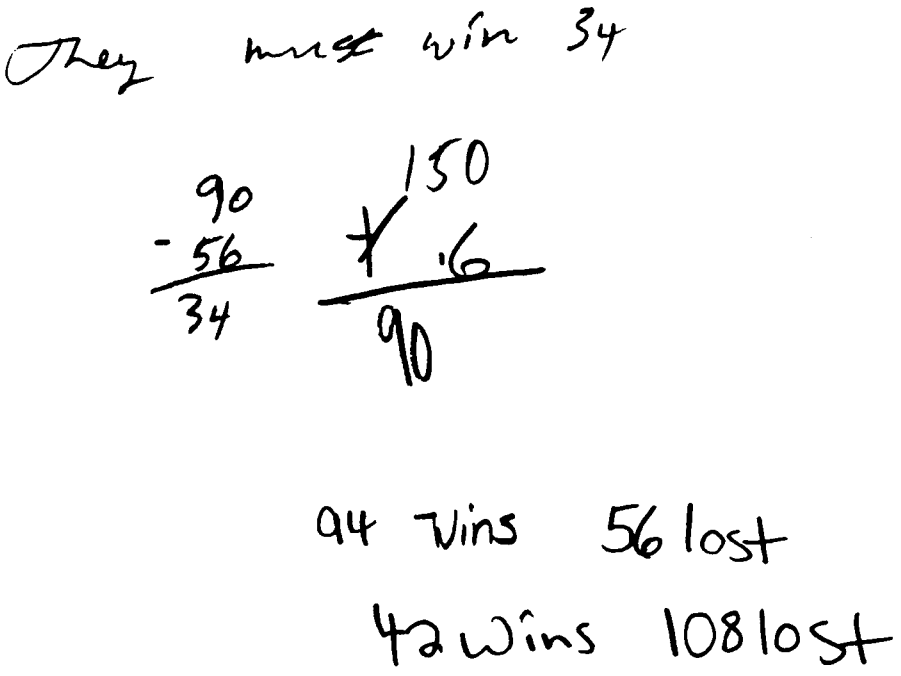
150 games × 60% = 90 games need to be won to have .6000 average

need to win already won
90 - 42 = 48 games more have to be won.

Score Point: 2

This response gives the correct number of games left to be won (48) in Part A with work shown, but does not do Part B.

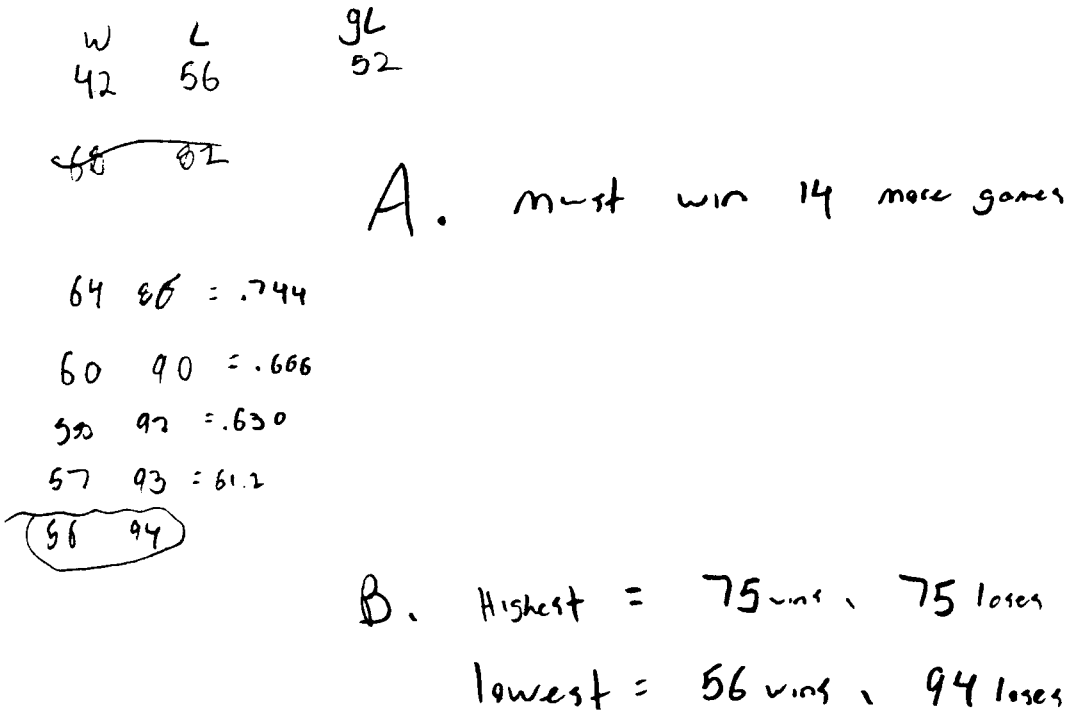
36g) Student Response 5



Score Point: 1

This response gives an incorrect answer for Part A (34) with an incorrect method used. The highest (94) and lowest (42) records are correctly given, but in a form other than decimals.

36h) Student Response 6

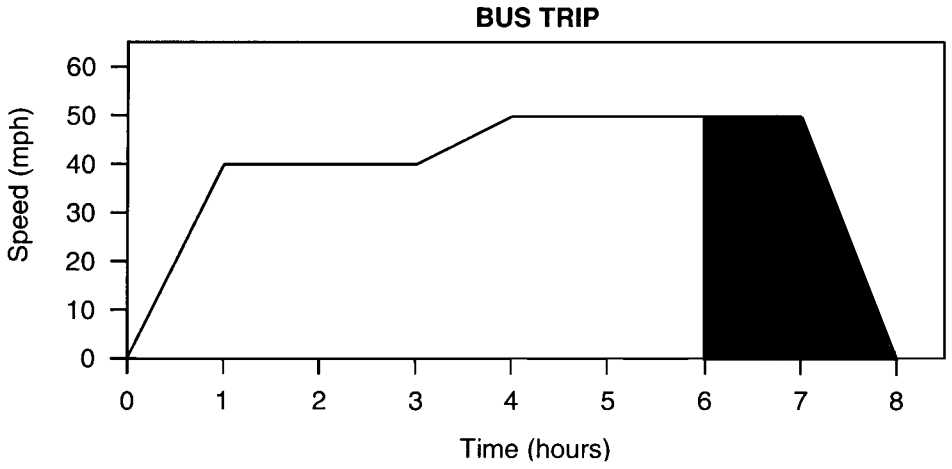


Score Point: 0

This response gives an incorrect answer for Part A (14) with an incorrect method used, and gives incorrect answers for Part B (75 wins, 56 wins).

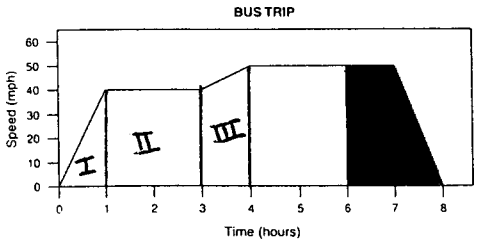
(3 Points)

37 The graph shows the average speed of a bus which carried a touring band during an 8-hour trip. The distance the bus traveled can be found by computing the area under the graph. For example, the distance traveled in the last two hours can be found from the area computed for the shaded portion on the graph.



Using the method described above, find the distance traveled in the first four hours. Provide the work that shows how you arrived at your answer.

37a) Sample Response



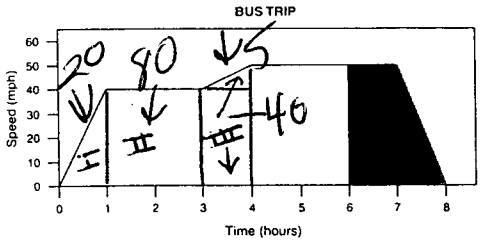
Method 1

Area I = ½ bh = ½ (1)(40) = 20

Area II = bh = 2 (40) = 80

Area III = ½ h(b1+b2) = ½ (1)(40+50) = 45

Total Distance: D = 20 + 80 + 45 = 145 miles



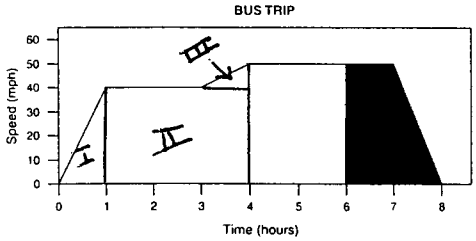
Method 2

Area I = ½ bh = ½ (1)(40) = 20

Area II = bh = 2 (40) = 80

Area III = bh + ½ bh = 1(40) + ½ (1)(10) = 40 + 5 = 45

Total Distance: D = 20 + 80 + 45 = 145 miles



Method 3

Area I = ½ bh = ½ (1)(40) = 20

Area II & Area III = bh + ½ bh = 3(40) + ½ (1)(10) = 120 + 5 = 125

Total Distance: D = 20 + 125 = 145 miles

Method 4

Subtract the areas of triangle A, rectangle B, and triangle C from the total area of the large rectangle.

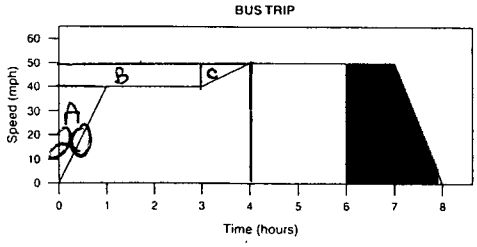
Triangle A: Area = ½ (1)(40) = 20

Rectangle B: Area = 10(3) = 30

Triangle C: Area = ½ (10)(1) = 5

Large rectangle: Area = 4(50) = 200

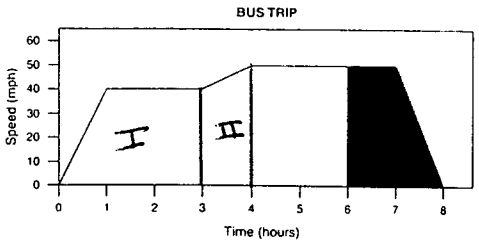
Total Distance: D = 200 – 20 – 30 – 5 = 145 miles



Method 5

Area I = ½ h(b1+b2) = ½ (40)(3+2) = 100

Area II = ½ h(b1+b2) = ½ (1)(40+50) = 45



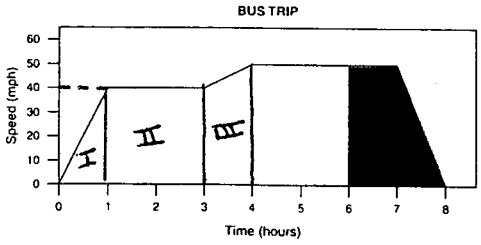
Method 6

Distance = Rate x Time ; for Area III, Rate = median speed

Area I = (40)(1)/2 = 20

Area II = (40)(2) = 80

Area III = (45)(1) = 45



37b) Scoring Rubric

3 Response contains correct distance (145 miles) and sufficient work is shown.

2 Response contains correct distance (145 miles) but not enough work is shown.

OR

The correct method is used to calculate the distance and sufficient work is shown, but a minor error results in an incorrect distance.

OR

Response contains the correct area for all but one of the sections of the graph and sufficient work is shown.

OR

The areas for all the sections of the graph are correct, but the total sum is incorrect.

1 Response contains correct distance (145 miles) but no work is shown.

OR

Response contains the correct area for only one of the sections of the graph and sufficient work is shown, provided that the graph is divided into more than two sections.

OR

Although the distance given is incorrect, work is shown in an attempt to calculate the distance and a logical answer results.

OR

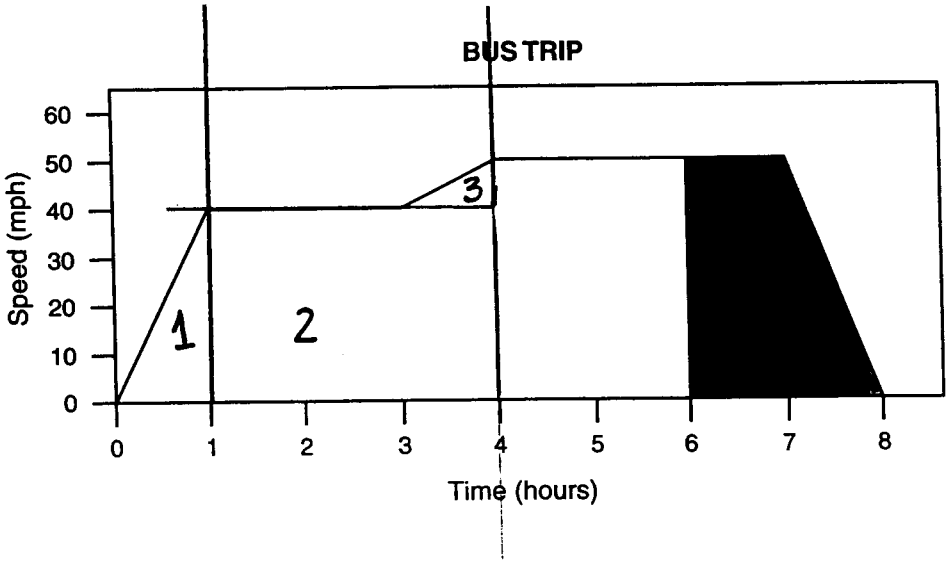
The student correctly finds the distance travelled for only the first two hours.

0 An incorrect distance is given, and no relevant work is shown.

OR

Due to a conceptual error, the student uses an incorrect method and merely adds the values found on the line of the graph (40+40+40+50) to get a total of 170.

37c) Student Response 1



Find the areas of all 3 regions then add them together

Region 1: $A = \frac{1}{2}(\text{base}) \times \text{ht.}$

$A = \frac{1}{2}(1) \times 40 = 20$ miles

Region 2: $A = l \times w$

$l = 40$ miles

$w = 4 - 1 = 3$ miles

$40 \times 3 = 120$ miles

Region 3: $A = \frac{1}{2}(\text{base}) \times \text{ht.}$

$= \frac{1}{2}(\text{base} = 50 - 40) \times (\text{ht} = 4 - 3)$

$= \frac{1}{2}(10) \times 1$

$= 5$

Add all regions together

$20 + 120 + 5 =$

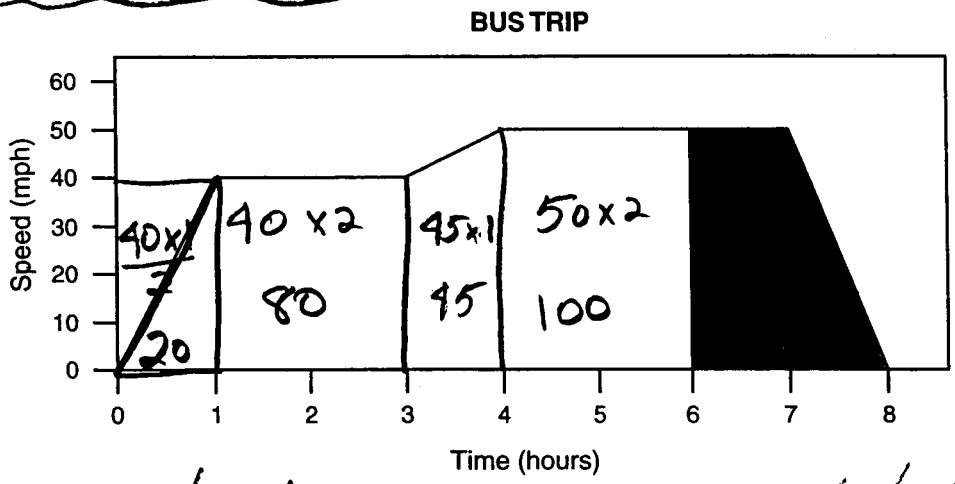
145 miles

Score Point: 3

This response contains the correct distance (145 miles) with sufficient work (Method 3).

37d) Student Response 2

Distance = Rate x Time



In order to find distance you must take the rate x the Time. for the first hour I set it equal to 40 mph. x 1 = 40 and then divided by 2 which equaled 20 miles for the first hour. The for the next 2 hours I took 40mph x2 to get 80 miles. for the 4th hour I took the medial speed which would be 45mph and X it by the time (1 hour) and got 45 miles.

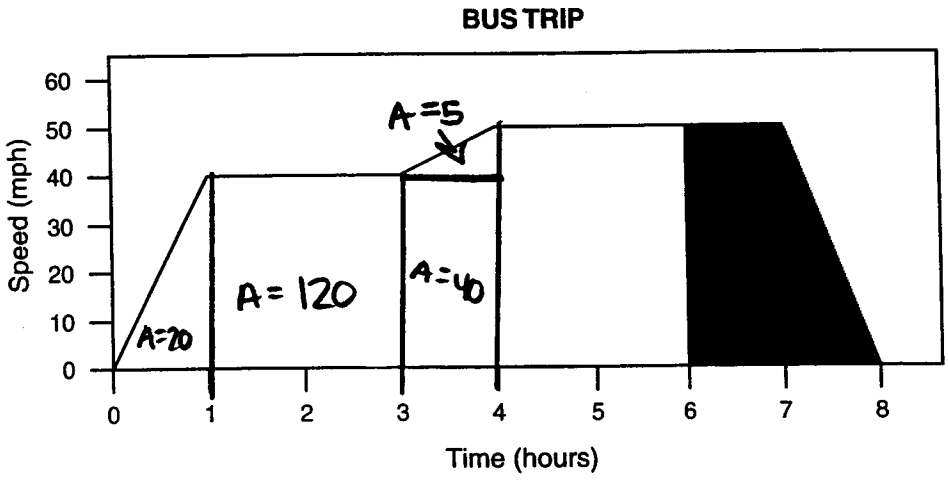
This is what the final equation would look like for the first 4 hours.

20 + 80 + 45 = 145 miles traveled in the first four hours of the trip.

Score Point: 3

This response contains the correct distance (145 miles) with sufficient work (Method 6).

37e) Student Response 3



$A_T = \frac{1}{2}(1)(40$

A=20

20 + 120 + 40 + 5 = 185 miles

$A_R = 3 \times 40$

A=120

$A_T = \frac{1}{2} B \times h$

$A_R = 1 \times 40$

A=40

$A_R = Bh$

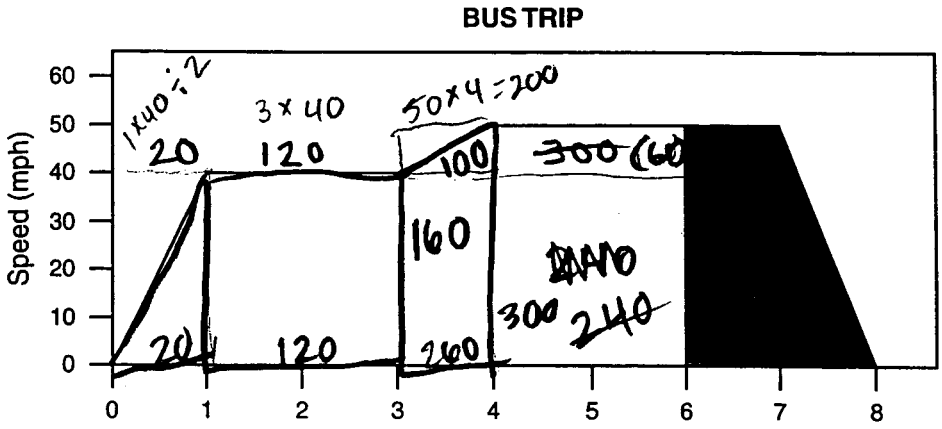
$A_T = \frac{1}{2}(1)(10)$

A=5

Score Point: 2

This response contains the correct area for all but one of the sections (120) and sufficient work is given (Rubric 2c).

37f) Student Response 4



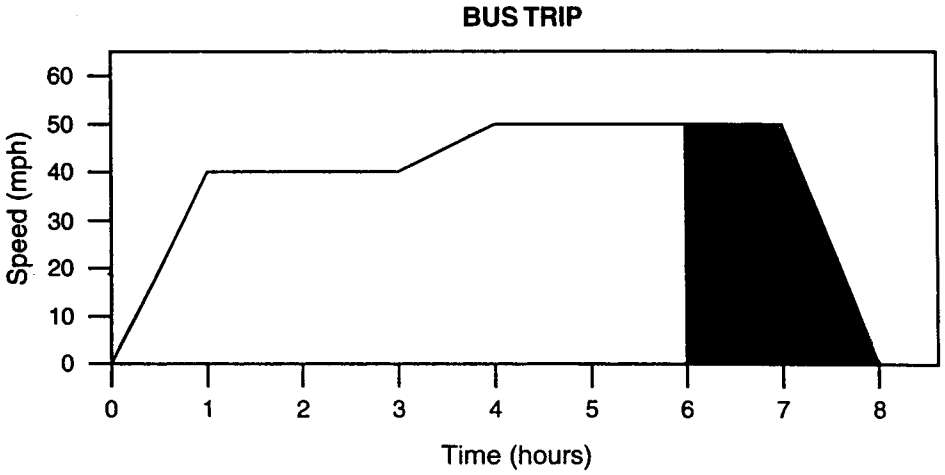
Distance = Rate x Time

The distance traveled is 400mi. Finding the area by breaking down each hour and then adding them together (shown above).

Score Point: 1

This response contains the correct area for the first section of the graph with sufficient work, and shows more than two sections (Rubric 1b).

37g) Student Response 5



First 4 hours =

Hours
1 2 3 4
40mph 40mph 40mph 50mph

170 miles in 4 hrs.

If you take the speed of the first 4 hours of the trip and add the times together you will get the total distance for the first 4 hours of the 8 hour trip.

Score Point: 0

This response gives an incorrect method and merely adds the values found on the line of the graph (40+40+40+50) to get a total of 170 (Rubric 0b).

Michigan Educational Assessment Program
Statewide Test Item Analysis
HST in Mathematics

Spring 2000

District: PUBLIC SCHOOL
Run Date: 09/06/2000

Multiple Choice

Item No.	Objective Code	Percent Answering by Response A	B	C	D	Omit/ Mult
Number						
21	NU	70*	19	5	5	0¤
23	NU	10	64*	4	21	0¤
29	NU	3	82*	13	2	0¤
Geometry and Measurement						
01	GE	3	13	5	79*	0¤
12	GE	6	10	22	62*	0¤
25	GE	7	32*	31	30	1
26	GE	43*	36	15	5	1
32	GE	45*	14	23	17	1
Data Analysis and Probability						
05	DA	4	4	6	85*	0¤
06	DA	26	7	61*	6	0¤
13	DA	4	8	80*	8	0¤
22	DA	10	69*	11	10	0¤
Algebraic Ideas						
03	AL	21	73*	3	3	0¤
04	AL	2	9	25	64*	0¤
17	AL	46	10	33*	11	0¤
19	AL	80*	10	8	2	0¤

Number Tested: 78212

Constructed Response

Item No.	Objective Code	Percent Receiving Number of Points 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	Percent Receiving Condition Codes A	B	C	D
Cross-Content														
35	CC	36	7	17	6	21					1	0	0	12
36	CC	22	3	4	2	12	3	7	4	32	2	0	0	9
37	CC	49	4	3	2	8	0	22			1	0	0	10

Condition Codes for the Constructed Response Items:

A Off-topic
B Illegible
C Written in language other than English
D Blank/refused to respond

The Objective Codes correspond to those used in the *Assessment Framework for the Michigan High School Proficiency Test in Mathematics* (1994).

NU = Number
GE = Geometry and Measurement
DA = Data Analysis and Probability
AL = Algebraic Ideas
CC = Cross-Content

Number:
understanding and using whole numbers, integers, fractions, decimals, ratios, and percents to solve real world mathematical problems

Geometry and Measurement:
measuring, building, drawing, visualizing, and using informal methods to solve geometric problems; drawing valid conclusions from information; using properties of shapes and relationships among shapes to solve geometric and measurement problems geometrically

Data Analysis and Probability:
describing, interpreting, and analyzing data; solving problems using charts, tables, and graphs; describing, interpreting, and analyzing simple statistical and probability experiments

Algebraic Ideas:
representing and solving problems physically, graphically, verbally, and symbolically using models, variables, expressions, number sentences, and simple arrays; analyze and describe mathematical functions and relations

Cross-Content:
These items assess student mathematical reasoning across the four content strands described above. The cross-content score is based on the student's ability to communicate, reason, and present mathematical solutions to problems.

Omit/Mult = Omits and Multiple Responses
¤ Number of students present rounds to zero